

$$\textcircled{1} \text{ a) } x^2 - y^2 = 2xy$$

$$2x - 2y \frac{dy}{dx} = 2x \cdot \frac{dy}{dx} + y \cdot 2$$

$$2x - 2y = 2x \frac{dy}{dx} + 2y \frac{dy}{dx}$$

$$2x - 2y = \frac{dy}{dx} (2x + 2y)$$

$$\frac{2x - 2y}{2x + 2y} = \frac{dy}{dx}$$

$$\text{b) } x^3 + xy + y^3 = 4$$

$$3x^2 + x \frac{dy}{dx} + y \cdot 1 + 3y^2 \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-y - 3x^2}{x + 3y^2}$$

$$\text{c) } y^{-1} + x^{-1} = 2$$

$$-y^{-2} \frac{dy}{dx} + -x^{-2} = 0$$

$$-\frac{1}{y^2} \cdot \frac{dy}{dx} - \frac{1}{x^2} = 0$$

$$-\frac{1}{y^2} \cdot \frac{dy}{dx} = \frac{1}{x^2}$$

$$\frac{dy}{dx} = -\frac{y^2}{x^2}$$

$$\text{d) } 3x^4 = (2xy - 1)^3$$

$$12x^3 = 3(2xy - 1)^2 \cdot (2x \frac{dy}{dx} + y \cdot 2)$$

$$\frac{12x^3 - 6y(2xy - 1)^2}{6x(2xy - 1)^2} = \frac{dy}{dx}$$

$$\textcircled{2} \text{ a) } y^2 = \frac{x^2 - 4}{x^2 + 4}$$

$$2y \frac{dy}{dx} = \frac{(x^2 + 4)(2x) - (x^2 - 4)(2x)}{(x^2 + 4)^2}$$

$$\frac{dy}{dx} = \frac{(x^2 + 4)(2x) - (x^2 - 4)(2x)}{(x^2 + 4)^2 \cdot 2y}$$

$$\left. \frac{dy}{dx} \right|_{(2,0)} = \frac{8 \cdot 4 - 0}{8^2 \cdot 0} \text{ undef.}$$

tangent:  $x = 2$

Normal:  $y = 0$

$$\text{b) } (x+y)^3 = x^3 + y^3$$

$$3(x+y)^2 \left(1 + \frac{dy}{dx}\right) = 3x^2 + 3y^2 \cdot \frac{dy}{dx}$$

$$3(0)^2 \cdot \left(1 + \frac{dy}{dx}\right) = 3(-1)^2 + 3(1)^2 \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = -1$$

normal  
tangent:  $y - 1 = x + 1$

tangent  
normal:  $y - 1 = -1(x + 1)$

plug  
in  $(-1, 1)$

→



$$\textcircled{3} \quad 4y^3 \frac{dy}{dx} = 2y \frac{dy}{dx} - 2x \quad \left\{ \begin{array}{l} \text{horiz tangent when } \frac{dy}{dx} = 0 \\ 2x = 0 \Rightarrow x = 0 \\ \text{when } x = 0 \rightarrow y^4 = y^2 \\ y^4 - y^2 = 0 \\ y^2(y^2 - 1) = 0 \\ y = 0; y = \pm 1 \end{array} \right.$$

Vert. tangent  $\frac{dy}{dx}$  undef

$$2y - 4y^3 = 0$$

$$2y(1 - 2y^2) = 0$$

$$y = 0 \quad y = \pm \frac{1}{\sqrt{2}}$$

$$\downarrow \quad \text{when } y = \pm \frac{1}{\sqrt{2}} \rightarrow \frac{1}{4} = \frac{1}{2} - x^2$$

$$x^2 = \frac{1}{4}$$

~~(0,0)~~

$$x = \pm \frac{1}{2}$$

So vertical @  $(\pm \frac{1}{2}, \pm \frac{1}{\sqrt{2}})$  (4 points)

@ (0,0)  $4y^3$  approaches 0 much faster than  $2x$  or  $2y$  so

$$\frac{dy}{dx} \rightarrow \frac{2x}{2y} = \pm \frac{2}{2} = \pm 1 \quad \left( \text{b/c graph approaches } y^2 = x^2 \rightarrow y \approx \pm x, \text{ near } 0 \right)$$

So graph is neither horiz. or vert. @ (0,0)



④ a)  $y = x^4 - 3x^2 + 4$   
 $y' = 4x^3 - 6x$   
 $y' = 0 \rightarrow 2x(2x^2 - 3) = 0$   
 $x = 0$  ;  $x = \pm\sqrt{\frac{3}{2}}$   
 outside domain

$y(-1) = 2$  Abs. min  
 $y(0) = 4$  Abs. max  
 $y(1) = 2$

b)  $y = x^3 + 6x^2 + 9x + 3$   
 $y' = 3x^2 + 12x + 9$   
 $y' = 0 \rightarrow 3(x^2 + 4x + 3) = 0$   
 $x = -1, -3$

$y(-4) = -64 + 96 - 36 + 3 = -1$   
 $y(-3) = -27 + 54 - 27 + 3 = 3$   
 $y(-1) = -1 + 6 - 9 + 3 = -1$   
 $y(0) = 3$   
 Abs min: -1, Abs Max 3

⑤ a)  $y = x^4 - 10x^2 + 9$   
 $y' = 4x^3 - 20x = 4x(x^2 - 5)$   
 $x = 0$  ;  $x = \pm\sqrt{5}$

$y$  increasing for  
 $x \in (-\sqrt{5}, 0) \cup (\sqrt{5}, \infty)$   
 $y$  decreasing for  
 $x \in (-\infty, -\sqrt{5}) \cup (0, \sqrt{5})$

$y'$  sign chart:  
 $\begin{array}{c} - & + & - & + \\ \hline -\sqrt{5} & 0 & \sqrt{5} \end{array}$

b)  $y = \frac{-x}{x^2 + 4}$   
 $y' = \frac{(x^2 + 4)(-1) - (-x)(2x)}{(x^2 + 4)^2}$   
 $y' = \frac{-x^2 - 4 + 2x^2}{(x^2 + 4)^2} = \frac{x^2 - 4}{(x^2 + 4)^2}$

$y$  increasing on  
 $x \in (-\infty, -2) \cup (2, \infty)$   
 $y$  decreasing for  $(-2, 2)$

$y' = 0$  when  $x = \pm 2$

$y'$  undef  $\emptyset$

$y'$  sign chart:  
 $\begin{array}{c} + & - & + \\ \hline -2 & 2 \end{array}$